Introduction (1)

- IAU/IAG Joint Working Group on Theory of Earth Rotation: Sub-Working Group 1 “Precession/Nutation”
- Chair: J. Getino, Spain
- Members (16):
  - Y. Barkin*, Russia; N. Capitaine, France; V. Dehant*, Belgium; A. Escapa*, Spain; J. Ferrándiz*, Spain; M. Folgueira*, Spain; A. Gusev, Russia; R. Gross, USA; T. Herring, USA; CL. Huang*, China; J. Müeller*, Germany; Y. Rogister, France; H. Schuh, Germany; J. Souchay*, France; V. Zharov, Russia; J. Vondrák*, Czech Republic
- Correspondent members (2):
  - G. Kaplan*; USA; S. Urban*, USA
- Chairs of SWG 2 & 3:
  - A. Brzeziński*, Poland; R. Heikelmann, Germany
(*) Contributors to this report
Introduction (2)

- Continues former report at EGU 2014 (http://web.ua.es/wgther)
- Here, we focus on the following potential actions, having in mind the proximity of next General Assemblies of IAG (22 June, Prague) and IAU (2 August, Honolulu):
  - Feasible enhancements of current precession/nutation model by
    - Completing the changes needed to get full consistency between the new precession theory and the nutation one
    - Clarifying nomenclature of the involved models
  - Future improvements of the models:
    - Accounting for different effects that provide contributions above or near the 10 μas level and might play a role for observational demands and/or geophysical interpretation or better consistency
    - Some of them requires a careful analysis, since they could entail a change in the basic Earth model considered in IAU2000A nutation

Current precession/nutation model (1)

  - IAU 2000A (0.2 mas level) or IAU 2000B (1 mas level)
  - Nutational part is a clear improvement over IAU 1980 nutation model
  - Precessional part is basically that of IAU 1976 (Lieske et al. 1977), updated with corrections to precession rates
  - Encouraged the development of new expressions for precession consistent with the IAU 2000A

  - Precession component of IAU 2000A replaced by P03 precession theory
Current precession/nutation model (2)

- At the highest levels of precision, the replacement of the IAU 2000 precession part by P03 is not direct:
  - Some nutation terms must be corrected to keep consistency (Capitaine & Wallace 2006), due to changes of some relevant parameters derived from P03.

- The main adjustments ($H$ is almost identical) are due to:
  - **The inclusion of $J_2$ rate in P03:**
    - It contributes to Poisson terms (mixed secular) in nutation both in longitude and obliquity (Capitaine & Wallace 2006). In addition, it also originates some out of phase terms (Escapa et al. 2013).
  
  - **The change in the value of the obliquity $\varepsilon_0$ in P03:**
    - Affects nutations in longitude through a scale factor $\sin (\varepsilon_0)$, accounted by Capitaine & Wallace (2006).
    - Changes all the reference rigid Earth nutation amplitudes in longitude and obliquity via Kinoshita’s functions $B(\varepsilon_0)$ (Escapa et al. 2013).

Current precession/nutation model (3)

Numerically the total adjustments are (shown > 1 μas):


<table>
<thead>
<tr>
<th>I</th>
<th>I’</th>
<th>F</th>
<th>D</th>
<th>Ω</th>
<th>$t*d\psi$</th>
<th>$t*d\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>47.8</td>
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<td>0</td>
<td>0</td>
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<td>-0.6</td>
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<td>3.5</td>
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<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

- **$J_2$ rate**, out of phase terms: Escapa et al. 2013 (new)

<table>
<thead>
<tr>
<th>I</th>
<th>I’</th>
<th>F</th>
<th>D</th>
<th>Ω</th>
<th>$d\psi$</th>
<th>$d\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1.4</td>
<td>-0.8</td>
</tr>
</tbody>
</table>
Current precession/nutation model (4)

- $\varepsilon_0$ change, **global rescaling**: Capitaine & Wallace 2006, Escapa et al. 2013 – high agreement

<table>
<thead>
<tr>
<th>I</th>
<th>I'</th>
<th>F</th>
<th>D</th>
<th>$d\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-8.1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

- $\varepsilon_0$ change, **consistency of rigid solution (new)**: Escapa et al. 2013

<table>
<thead>
<tr>
<th>I</th>
<th>I'</th>
<th>F</th>
<th>D</th>
<th>$d\psi$</th>
<th>$d\varepsilon$</th>
<th>$t^*d\psi$</th>
<th>$t^*d\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-7.5</td>
<td>0.8</td>
<td>-8.1</td>
<td>-</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

- $\varepsilon_0$ change, **total correction**: rescaling + rigid consistency ($\mu$as, $\mu$as/cJ):

<table>
<thead>
<tr>
<th>I</th>
<th>I'</th>
<th>F</th>
<th>D</th>
<th>$d\psi$</th>
<th>$d\varepsilon$</th>
<th>$t^*d\psi$</th>
<th>$t^*d\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-15.6</td>
<td>0.8</td>
<td>-8.1</td>
<td>-</td>
</tr>
</tbody>
</table>

- **Open question (to be discussed):**
  - Should the current numerical values of the adjustments to MHB2000 nutations (Capitaine & Wallace 2006) be completed?

Current precession/nutation model (5)

- **Hence, nowadays some combinations in use** are (Urban and Kaplan 2011):
  - (1) P03 (prec., IAU 2006) + MHB2000 (nut. part, IAU 2000A)

- (2) is considered in IERS Conventions 2010, Standards of Fundamental Astronomy (SOFA), and Explanatory Supplement to the Astronomical Almanac

- **As recognized by Urban and Kaplan (2011),** there are used different names to designate the former combinations, e.g.:

<table>
<thead>
<tr>
<th>Comb.</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
</table>
| IERS | IAU 2006/2000A | IAU 2006/2000A
| SOFA | IAU 2006/2000A (suffix “00A”) | IAU 2006/2000A (suffix “06A”)

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SWG1 Report, Journées 2014
Current precession/nutation model (6)

Open questions (to be discussed):

- Should combination P03 (prec., IAU 2006) + MHB2000 (nut. part, IAU 2000A) + Adjustments to MHB2000 be officially supported by IAU/IAG JWG_ThER through some action?
- Should IAU/IAG JWG_ThER suggest or recommend a clear terminology for the models/algorithms in use, e.g., Kaplan (2009), Urban & Kaplan (2011), etc.?

Future improvements of the model (1)

- After IAU2000 adopted model on nutation (Mathews et al. 2000), scientific contributions related with SWG1 issues have been focused mainly on new second order effects
- Second order effects comprise terms arising from crossing first order contributions in the perturbation sense (mathematical), and also not modelled (or ill modelled) terms whose magnitude is small (physical)
- These effects provide corrections of the order of some tens of μas (or more):
  - Observational demands
  - Geophysical interpretation
  - Better precession-nutation consistency
Future improvements of the model (2)

- Next, we list some topics contributed by the members and correspondents of this subgroup
- There is an extended description of some of them that will be added (with permission of the contributors) to this presentation
- We have listed them following a chronological order as they contacted the chair of SWG1
- For brevity, it is just indicated the name of the member/correspondent of the SWG1, although some works are the result of a cooperation with other colleagues
- Several issues are presented in different talks of these Journées, we encourage you to attend them to obtain more details of the research directly from their authors
- There are other talks also of interest for this SWG1

Future improvements of the model (3)

- J. Souchay (proposals):
  - To study the influence of the Moon when considering it as a triaxial, not pointlike object: old computations indicated an effect at the μas level
  - To study the precession-nutation in primary ages of the solar system, when the Moon was considerably closer to the Earth: It should gather quite a good number of specialists, combining rigid and non rigid aspects

- C. Huang:
  - Earth nutation and its coupling with the magnetic field: in a displacement field approach the contribution of the Electromagnetic Coupling to FCN is one order of magnitude smaller than in MHB2000
  - New theory of Earth rotational modes (app. to FCN): by using the Galerkin method and developing a linear operator and a new multiple layer spectral method, it was obtained in a first result a period of 435 sidereal days for the FCN (Session 4, tomorrow 09:00-10:20, Do we need various assumptions to get a good FCN? - A new multiple layer spectral method by Huang & Zhang)
  - A generalized theory of the figure of the Earth interior: using a new potential/figure theory and real surface layers data, obtaining a value for the dynamic flattening $H = 1/306.88$
Future improvements of the model (4)

- **J. Müller:**
  - Nutation determined from only Lunar Laser Ranging (LLR) data: fit of luni-solar nutation coefficients from 44 years of LLR data for nutation periods of 18.6 years, 9.3 years, 1 year, 182.6 days and 13.6 days, by using different realizations of precession/nutation for ICRS-ITRS transformation

- **J. Vondrák:**
  - Numerical integration of Brzeziński’s broad-band Liouville equations: applied to estimate atmospheric and oceanic excitation of nutation. It is documented that the effect is significant, especially at annual and semi-annual periods, the amplitudes reaching 0.1mas (*Session 4, today 16:00-17:40, Geomagnetic excitation of nutation by Ron & Vondrák*)

Future improvements of the model (5)

- **Y. Barkin:**
  - Study of the perturbed rotational motion of the Earth: construction of a first-order perturbations theory in Andoyer variables and for the projections of the angular velocity of rotation of the planet caused by the weak variation of the mass geometry and the components of the angular momentum of the relative motion of the particles of the planet

- **V. Dehant & M. Folgueira:**
  - Topographic coupling at core-mantle boundary in rotation and orientation changes of planets: four coupling mechanisms (topographic, viscous, gravitational and electromagnetic torques) are computed to revise their relative importance in the terrestrial bodies, with particular emphasis on the topographic coupling (*Session 4, today 14:00-15:30, Refinements on precession, Nutation, and Wobble of the Earth by Dehant*)

- **A. Brzeziński (proposal):**
  - Convenience of splitting up the scope of SWG1 and SWG2 based on geophysical mechanism: the geophysical excitations of nutations (long period) should be considered by SWG2, while modeling the librations (astronomical) in polar motion by SWG1
Future improvements of the model (6)

- **A. Brzeziński:**
  - Atmospheric and Oceanic Excitation of the Free Core Nutation Estimated from Recent Geophysical Models
  - On estimation of the high frequency geophysical signals in Earth rotation by complex demodulation *(Session 4, today 14:00-15:30, On application of the complex demodulation procedure for monitoring Earth rotation: comparison with the standard approach using the long periodic EOP components estimated from VLBI data analysis by the VieVS CD software by Brzeziński, Wielgosz, & Boehm)*

- **A. Escapa:**
  - Direct effects of the rotation of the inner core: due to the differential rotation of the inner core, providing contributions to the nutations at the µas level (not in IAU2000)
  - Influence of the triaxiality of the Earth: currently as corrections to polar motion. There is no considered the triaxiality of the core, neither its indirect effects on the nutations, although nowadays there is some work on this issue (Chen & Shen 2010, *Poster session, Triaxial Earth’s rotation: Chandler wobble, free core nutation and diurnal polar motion by Sun & Shen*)

Future improvements of the model (7)

- **J. Getino:**
  - New perturbation technique to integrate higher orders in the Earth rotation theory: by using a matrix formulation of the equations of motion, the dynamical variables are gathered together in two matrix variables, while the considered model is represented by the matrices of the system. Second order analytical solutions are obtained in a systematic way. The inclusion of new effects is reduced to the redefinition of the matrices of the system

- **J. M. Ferrándiz:**
  - Consistency among nutation and precession theories: second order and tidal effects of the non-rigid Earth stemming from the nutation model also contribute to the precession rates, so the precession model is also affected by complex non-rigid nutation interactions *(Session 4, today 16:00-17:40, Effects of the tidal mass redistribution on the Earth rotation by Ferrándiz, Baenas, Escapa, & Getino)*
Future improvements of the model (8)

Open questions (to be discussed):
- The integration of previous described effects into a single consistent theory present a complex scenery, e.g.:
  - Could IAU2000A basic (symmetric) Earth model be preserved or should we move to another more sophisticated model?
  - How to homogenize their theoretical analysis to “plug” them into a global model?
  - How much of this task can be carried out in this term?
Annex

Report on activities of the Sub-Working Group 1
"Precession/Nutation" of the IAU/IAG
Joint Working Group on Theory of
Earth Rotation

Extended description of some of the issues contributed by the members and correspondent members of SWG 1
Jean Souchay, France  
(jean.souchay@obspm.fr)

Proposals

- Study the influence of the Moon when considering it as a triaxial, not pointlike object (fast calculations carried out long time before should indicate an effect at the level of a few microarcseconds).

- Study the precession-nutation in primary ages of the solar system, when the Moon was considerably closer to the Earth. => It should gather quite a good number of specialists among us (rigid and non rigid aspects).
Earth nutation and its coupling with magnetic field

Free core nutation (FCN) is a rotational modes of the earth with fluid core, and it reflects the physics of the fluid outer core and influences the annual nutation with strong resonance. All traditional theoretical methods produce FCN period near 460 days with standard earth model (like PREM), while observations (VLBI + SG tides) say it is near 430±1 days. In order to fill this big gap, astronomers and geophysicists give various assumptions, e.g., increasing core-mantle-boundary (CMB) flattening by about 5%, a strong coupling between nutation and geomagnetic field near CMB (EMC), etc. In the current IAU2000 nutation model (MHB model), EMC mechanism was used to fill this gap and they got ‘better’ result than other nutation models.

We studied nutation amplitudes and FCN period in a displacement field approach in which the magnetic field influence is incorporated directly in the motion equation and in the boundary conditions. A new strategy to compute nutations is established. Although using the same parameters related to the magnetic field and electric conductivity as in MHB model, the result shows that the EMC contribution to FCN is one order of magnitude smaller than MHB model, i.e., EMC is too small to explain the gap of FCN period as MHB model stated. (Huang et al., 2011). It means that a new nutation theory/model is needed, as it becomes the key task of the new IAU/IAG joint workgroup ‘Theory of Earth Rotation (WG_ThER)’

New theory of earth rotational modes and its application to the study of free core nutation (FCN)

Since 2006, we started to use Galerkin method to study the earth rotational modes, and developed a liner operator method and a new multiple layer spectral method and applied them to the computation of normal modes. The combination of these two new methods can solve not only one order ellipsoid but also irregular asymmetric 3D earth model. Our primary result of the FCN period is 435 sidereal days without any unproved assumptions like mentioned above. (Zhang & Huang, 2009, 2014abc)
A generalized theory of the figure of the Earth interior and its application to the Study of global dynamical flattening

The Darwin - de Sitter second-order theory, comparing to the Clairaut first-order theory, is considered as standard theory of equilibrium figures of the Earth interior, though there are also some other higher-order or more general development such as Denis (1989). However, all these theories cannot deal with the contribution from the anti-axial-symmetric mass distribution, i.e. the spherical terms of non-zero order and odd degree. But the fact is that the inhomogeneous and anti-symmetry of the outermost crust, topography and oceanic layers are too big to ignore.

Since 2007, we have developed a new generalized theory to obtain the equilibrium figures interior to fully third-order accuracy, in which all the odd degree and non-zero order terms are included in the spherical harmonic expansion. Both the direct and indirect contribution of the anti-symmetric crust layer can then be included in this theory.

Using our new potential/figure theory and real surface layers data, we re-calculate the geometrical flattening profile of the Earth interior and, as a by-product, the global dynamic flattening (H), which is an important quantity in research of rotating Earth and is related with precession constant, is obtained (1/306.68). The significant difference (previously 1.1%) between the value of H from accurate precession observation and that from traditional theory is reduced, by about 2/3, to 0.38% (Liu & Huang, 2009) and improved again to 0.22% (Huang & Liu, 2013).

Furthermore, this new theory is applied to derive the non-spheroidal figure of the core-mantle boundary and then its contribution to the free core nutation (FCN) is also discussed.
Coupling between geomagnetic field and nutation (EMC)

Tab.: Contributions of EMC at CMB to nutation (μas) & FCN

<table>
<thead>
<tr>
<th>periods (day)</th>
<th>pro-ip (μas)</th>
<th>pro-op (μas)</th>
<th>retro-ip (μas)</th>
<th>retro-op (μas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6798.384</td>
<td>+4(+9,+37)</td>
<td>-4(-9,-29)</td>
<td>-20(-83,-328)</td>
<td>+20(+83,+249)</td>
</tr>
<tr>
<td>365.260</td>
<td>0(0,-3)</td>
<td>0(0,+3)</td>
<td>-40(-68,-450)</td>
<td>+40(+69,+411)</td>
</tr>
<tr>
<td>182.621</td>
<td>±5(\pm 14,\pm 61)</td>
<td>±5(-14,-47)</td>
<td>±2(-3,-16)</td>
<td>±2(\pm 3,\pm 12)</td>
</tr>
<tr>
<td>13.661</td>
<td>0(0,+2)</td>
<td>0(0,-1)</td>
<td>0(/,0)</td>
<td>0(/,0)</td>
</tr>
</tbody>
</table>

FCN without EMC = -432.34 solar day (-455.57 sidereal day)

FCN with EMC = -431.96 solar day (-454.39 sidereal day)

change in the FCN period = 0.38 solar day (1.18 sidereal day)

- FCN is a normal mode of the earth as the rotating axes of the FOC and of the mantle don’t coincide.
- FCN depends on the physics of the FOC, mantle & core-mantle-boundary. It influences strongly the retro-annual nutation due to its resonance, so it is a key parameter & interesting topic.
- The calculated period of FCN from traditional theory differs largely from the high-precision observation.
- We developed an integrated Galerkin method and spectral element method that can study any antisymmetric earth without GSH.
- These methods are applied on the computation of FCN period from PREM earth without any assumption (eg., extra flattening at CMB, magnetic & viscous couplings at CMB, etc.).
- Our result is 435 sid. day !

Study of Free-core-nutation (FCN)

- No fluid OC? ➔ No FCN !

<table>
<thead>
<tr>
<th>Obs.(VLBI+SG)</th>
<th>430±1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretically Calculated</td>
<td>458-465</td>
</tr>
<tr>
<td>This work</td>
<td>435</td>
</tr>
</tbody>
</table>

(Zhang & Huang, 2014a,b, submitted)
A generalized theory of figure of the earth

Clairaut, 1743
Darwin, 1899; De Sitter, 1924
Denis, 1985-2006

\[ r = s \left[ 1 + s_2 P_2 + s_4 P_4 + s_6 P_6 \right] \]

\[ r(s, \theta, \phi) = s \left[ 1 + \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{H_n^m(s) Y_n^m(\theta, \phi)}{1/H} \right] \]

(Liu & Huang, 2009)

A generalized theory of figure of the earth; application to the MoI & global dynamic flattening (H)

Precession Obs.: 305.5

<table>
<thead>
<tr>
<th></th>
<th>A(10^{27} kg m^2)</th>
<th>B(10^{27} kg m^2)</th>
<th>C(10^{27} kg m^2)</th>
<th>1/H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>8.0115651</td>
<td>8.0115651</td>
<td>8.0376170</td>
<td>308.52</td>
</tr>
<tr>
<td>theory + PREM</td>
<td></td>
<td></td>
<td></td>
<td>+1.1%</td>
</tr>
<tr>
<td>This work:</td>
<td>8.0036624</td>
<td>8.0035255</td>
<td>8.0293482</td>
<td>311.77</td>
</tr>
<tr>
<td>direct effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This work:</td>
<td>8.0112300</td>
<td>8.0114003</td>
<td>8.0375719</td>
<td>306.12</td>
</tr>
<tr>
<td>direct + indirect effects</td>
<td></td>
<td></td>
<td></td>
<td>+0.2%</td>
</tr>
<tr>
<td>EGM2008[17]</td>
<td>8.0100829</td>
<td>8.0102594</td>
<td>8.0364807</td>
<td>305.46</td>
</tr>
</tbody>
</table>

(Liu & Huang, 2009; Huang et al., 2013)
Nutation determined from only LLR data

**Initials:** precession and nutation according to IAU Resolution 2006 and IERS Conventions 2010

Use of **different realizations** of precession/nutation for ICRS-ITRS transformation

**Fit** of luni-solar nutation coefficients from 44 years of LLR data for nutation periods of 18.6 years, 9.3 years, 1 year, 182.6 days, (13.6 days), i.e. estimation of various $A', A'', B', B''$ of

$$\Delta \psi = \sum_{i=1}^{5} (A_i + A_i') \sin (ARG) + (A_i + A_i') \cos (ARG)$$

$$\Delta \varepsilon = \sum_{i=1}^{5} (B_i + B_i') \cos (ARG) + (B_i + B_i') \sin (ARG)$$

$$ARG = \sum_{i} N_i F_i \quad N_i : \text{multiplier, } F_i : \text{Delaunay parameters}$$

### Results - example

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-17206,42</td>
<td>2.70 ± 0.20</td>
<td>5.21 ± 0.25</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>9205,23</td>
<td>-0.48 ± 0.10</td>
<td>-1.32 ± 0.11</td>
<td></td>
</tr>
<tr>
<td>A'</td>
<td>3.34</td>
<td>-6.62 ± 0.12</td>
<td>-3.46 ± 0.21</td>
<td></td>
</tr>
<tr>
<td>B'</td>
<td>1.54</td>
<td>-2.29 ± 0.09</td>
<td>-2.19 ± 0.10</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>-1317,09</td>
<td>-2.38 ± 0.08</td>
<td>-1.69 ± 0.11</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>573,03</td>
<td>0.25 ± 0.05</td>
<td>0.15 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>A''</td>
<td>-1.37</td>
<td>1.80 ± 0.07</td>
<td>1.85 ± 0.09</td>
<td></td>
</tr>
<tr>
<td>B''</td>
<td>-0.46</td>
<td>0.23 ± 0.05</td>
<td>0.22 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>207,46</td>
<td>0.45 ± 0.11</td>
<td>0.85 ± 0.18</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>-89.75</td>
<td>-0.15 ± 0.07</td>
<td>-0.13 ± 0.08</td>
<td></td>
</tr>
<tr>
<td>A''</td>
<td>-0.07</td>
<td>-1.50 ± 0.12</td>
<td>-0.97 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>B''</td>
<td>-0.03</td>
<td>-0.87 ± 0.08</td>
<td>-1.35 ± 0.09</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>147,59</td>
<td>-2.91 ± 0.10</td>
<td>-0.51 ± 0.16</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>7.39</td>
<td>0.55 ± 0.06</td>
<td>0.01 ± 0.07</td>
<td></td>
</tr>
<tr>
<td>A''</td>
<td>1.12</td>
<td>-2.30 ± 0.09</td>
<td>-0.06 ± 0.11</td>
<td></td>
</tr>
<tr>
<td>B''</td>
<td>-0.19</td>
<td>-0.29 ± 0.05</td>
<td>-0.02 ± 0.05</td>
<td></td>
</tr>
</tbody>
</table>

LLR 1: precession according to Fukushima (2003) and Williams (1994)
LLR 2: precession according to P03, Capitaine et al. (2003)

L. Biskupek, J. Müller, IfE, University of Hannover
Discussion of the results

Differences in estimated corrections to the MHB2000 nutation model also depend on the implementation of precession/nutation (correlations change …)

Realistic accuracy of nutation coefficients from LLR about 0.3 mas in obliquity and 0.5 mas in longitude

Largest differences in longitude components

Large correlation of 18.6 and 9.3 year periods

Major problems in LLR are the unevenly distributed data (gaps in time series, orbit coverage, only few sites, weather, less accuracy in early years, …)

Future plan: Joined analysis of LLR and VLBI
Numerical integration of Brzezinski’s broad-band Liouville equations is used to estimate atmospheric and oceanic excitation of nutation. It is documented that the effect is significant, especially at annual and semi-annual periods, the amplitudes reaching 0.1mas. The atmosphere and oceans are capable of exciting the free core nutation that would otherwise be damped. However, different models of the atmosphere/oceans (NCEP+ECCO, ERA+OMCT) give slightly different results. The best agreement with celestial pole offsets, observed by VLBI, is achieved if NCEP atmosphere with IB correction is used. Additional schematic excitations, introduced at the epochs of geomagnetic jerks, improve the agreement between the integrated and observed celestial pole offsets remarkably, as demonstrated by Figs. 1 and 2.

Figure 1 Observed and excited celestial pole offsets, NCEP with IB correction.

Figure 2 Observed and excited celestial pole offsets, NCEP with IB correction plus additional excitations at the epochs of geomagnetic jerks.
References:
Yuri Barkin, Russia (barkin@inbox.ru)

Study of the perturbed rotational motion of the Earth and the Moon.

Y. Barkin, H. Hanada, M. Barkin, J. Ferrandiz

Introduction

In our previous studies the theory of rotation of the rotational motion of the rigid Moon have been developed (Barkin, 1987, 1989). Performed the first development of the theory of rotational motion of the Moon with the ellipsoidal liquid core (Barkin Yu., Barkin M., H. Hanada et al., 2012, 2014). Performed the first study of the rotational motion of the Earth with variable geometry mass obtained by space geodesy methods (M. Barkin, 2014). Carried out preliminary studies of the dynamic role of the moveable gravitating core (center of mass) of the Earth in geodetic changes of the planet (Barkin, Shatina, 2004; Barkin, 2005), tectonic polar restructuring of the Earth, in a polar inversion formation of supercontinents (Bozhko, 1992; Bozhko Barkin, 2000 and others.), in celestial mechanics and geodynamics (Barkin, 2001).

Some of geodynamic phenomena and planetary geophysical processes have been obtained an explanation from mechanical and energy point of view. In particular, the secular variations of gravity at the leading gravity stations (Barkin, 2010), secular effects in the motion of the poles of the Earth and the tidal acceleration of the Earth's rotation (Yu. Barkin, 2010; M. Barkin, 2014), the secular variations of the mean sea level in the northern and southern hemispheres (Barkin, 2011), the secular variations of the coefficients of the geopotential (Barkin, 2001), fundamental phenomena in geotectonics (Goncharov et al., 2012), and others.

In our studies already performed the first study on the improvement of the development of the analytical theory of the Earth's rotation and the theory of the physical libration of the Moon for their models with ellipsoidal liquid core and with an outer shell with variable mass distribution. These results will be published in monography (Barkin, 2014). In the future, we plan to continue these studies, involving more sophisticated models of the Earth and the Moon and with the application of new analytical methods for constructing theories of the rotational motion of the celestial bodies. Scheduled to perform the following studies.
1 Studies of the rotational motion of the Earth.

1.1. Development of a new approach to the study of the perturbed rotational motion of the planet as a result of changes in its mass geometry and the angular momentum of the relative motion of its particles on the basis of new forms of the equations of motion in the variables Andoyer and action-angle.

The study of the rotational motion of the Earth as an isolated planet and study perturbations in the motion of its axis of rotation due to variations in the geometry of its mass and the variations of the components of the relative angular momentum. In this part of the work will be used in modern satellite data on the observed secular, annual and semi-annual and others variations of the geopotential coefficients and the components of the tensor of inertia of the Earth.

1.2. An approximate analytical solution for the perturbed rotational motion of a variable celestial body.

Construction of the first-order perturbations in these variables and for the projections of the angular velocity of rotation of the planet caused by the weak variation of the mass geometry and the components of the angular momentum of the relative motion of the particles of the planet.

The novelty of the approach lies in the fact that the unperturbed rotational motion adopted Euler free motion of the axially symmetric planet (1 model) and the free rotational motion of Euler triaxial planet (with unequal principal moments of inertia, model 2). In the unperturbed rotational motions of the planet (of its poles) to take into account the elastic properties of the planet and its deformation caused by its rotation. Similar unperturbed motion can be termed Chandler - Eulerian motion. In particular, they describe the motion of the pole of the planet not with Euler period (for the Earth during this period 305 days), and with the observed period of Chandler (432 days). The angle between the mean polar axis of inertia and the angular momentum vector in the unperturbed motion is small and is about 0.25 seconds of arc.

1.3. Investigation of dynamic effects in the rotational motion of the Earth caused by temporal variations of the geopotential coefficients and their corresponding mass redistribution of the planet.

First of all be studied in detail the dynamic role of the secular variations, annual and semi-annual cyclic variations of the coefficients of the second harmonic of the geopotential and given a mechanical interpretation of the annual and semi-annual variations in pole position of the axis of rotation of the Earth. Assess the contribution of secular changes in change on
of the geopotential coefficients in observed phenomena: secular trend pole axis of rotation of the Earth and tidal acceleration of its axial rotation.

1.4. Tabulation of precession, nutation, and polar oscillations of Earth's rotation axis.

Building and tabulation of first-order perturbations caused by the gravitational moments from the Moon and the Sun at high-precise description of their orbital motion. It is assumed in these constructions perform analytically and get a first-order perturbation for model of solid non-spherical Earth and its model as a rigid body with variable outer shell to specify the redistribution of masses, which will be described on the basis of current satellite data on the secular, annual and semi-annual variations of the geopotential (including in particular tidal variations of planet). Obtain numerical estimates of the amplitudes of perturbations and present them in the form of tables, adopted in the theories of rotation of the Earth and the Moon. Particularly examine the role of planetary perturbations, perturbations of mixed type and quadratic time terms. Identify and explore new significant perturbations. Identify the components of these perturbations, which, due to dynamic effect of liquid ellipsoidal core.

2 Studies of the rotational motion of the Moon.

2.1. The study of forced physical libration of the Moon due to the second harmonic, third and higher harmonics (up to the sixth order).

In our previous works in the theory of the Moon's rotation it was considered a main part of the force function of the problem - the second harmonic of selenopotential. However, many aspects of the theory we have not yet considered. One of the major goals of our future study is to perform research on the construction and improvement of the theory of physical libration of the Moon, and to explore the role and contributions of the third and higher harmonics of selenopotential in forced and free libration of the Moon. Is expected to study the libration of the Moon, due to the gravitational influence of the Sun and the major planets. This study is an extremely concise, since it is associated with the construction of the auxiliary Poisson series of spherical functions which are given of certain functions of spherical geocentric coordinates of the Moon, Sun and planets.

2.2. Secular and mixed libration of the Moon, and their comparison with the data of the empirical theory.

In the future work, we will explore the role of planetary orbit perturbations of the Moon and get secular amendments to the constant amplitudes of libration (i.e., construct the tables of libration of the mixed type). An important role will be played by research to improve models of the Moon, on the design equations of rotational motion of its two-layer and three-layer models, the development of methods to construct an analytical theory of physical libration.
of the Moon. An important component and capacious work - is to build a Poisson series for spherical functions of the coordinates of the Moon on multiple arguments of the lunar theory.

2.3. Construction of new high-precision expansions of the force functions of the problem.

The work will continue to build power expansions of the problem of the rotation of the Moon with high-precision description of its orbital motion (the modern theory of LE 406). In our (total) task the force function takes into account the gravitational interaction of non-spherical Moon with the Earth, the Sun and the planets, and also takes into account the non-sphericity of the Earth. In our studies, a special Hamiltonian formalism based on the use of special forms of the equations of rotational motion of a solid and two-layer models of the Moon and the Earth has been developed (Barkin, 1987; Barkin et al., 2012). It is planned to generalize this approach to study the rotation of a three-layer Moon (and a four-layer Earth) (a liquid core and solid core, elastic mantle and outer shell with variable distribution of masses). As the main variables are selected variables Andoyer - Poincare.

To construct the analytic theory we have developed special methods of construction of quasi-periodic solutions of the equations of motion and to study their neighborhood. Conditionally - periodic solutions describe the forced libration, and the solution in their neighborhood describe the free and resonant libration of the Moon. As a basic model of rotation the two-layer model of the Moon is taken Poincaré model of simple fluid motion in a ellipsoidal cavity (the Poincaré problem).

For the three-layer model the Moon is considered as a system of three layers: a solid core, liquid core and mantle according to modern seismic model of these shells (Weber et al., 2011, etc..). For this model we will develop in detail the method of study of the perturbed rotational motion of a synchronous satellite with a solid core and ocean shell. And method for studying of the rotational motion of the Earth with a solid core and liquid core (Ferrandiz, Getino, 2000 -2007) will obtain a generalization.

To construct the necessary expansions of force functions in an intermediate stage of studies are used a mathematical approach and computer programs developed by S.M. Kudryavtsev (2007, 2009), for analytical operations with Poisson series. These programs and results on the construction of Poisson series in the theory of rotational motion of the Earth and the Moon will be developed. In first some improvements in the consideration of the third harmonic and higher order harmonics of selenopotential in real and complex forms.

To check the correctness of the theory, our results for the study of forced and free libration of the Moon with liquid ellipsoidal core compared with the modern empirical theory constructed by the classical method, but on the basis of about 40 - year laser observations of the Moon (Rambiaux, Williams, 2011), as well as with other theories.

The wide studies are being planned to develop a new geodynamic concept of forced relative oscillations of the shells of the Earth and the Moon (the mantle, liquid core and solid core), as well as other bodies in the solar system.
3 Kinematics and dynamics of the relative displacements and forced oscillations of the shells of the Earth (and Moon) and their correlation with variations of natural processes.

Another important field of planned research - this a study the kinematics and dynamics of systems of shell bodies with wide applications to the study of space-temporal patterns of planetary processes and their mutual correlations (primarily in relation to the Earth and the Moon). According to developing geomodel each celestial body - in fact, a system of celestial shells (core, mantle, etc.) that interact with each other gravitationally and subjected to cyclic gravitational influences from the surrounding celestial bodies. For the Earth's it is the Moon, the Sun and planets. This line of research opens a wide field of studies in celestial mechanics for explanation and understanding of planetary natural processes, their spatial and temporal properties. Earlier, before the advent of work in this direction, it was impossible to even think about explaining the energy and cyclical natural processes on the Earth.

References


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Proposal

As far as we are interested in scientific aspects of Earth rotation, a more adequate decomposition into polar motion and nutation is based on the excitation mechanism

- astronomical effects (due to the lunisolar and planetary torques upon the rotating Earth) are considered as nutation;
- geophysical effects (due to the mass and angular momentum exchanges between the solid Earth and its liquid envelopes) are considered as polar motion.

Our proposal is to follow the last decomposition in the discussion of the WG ThER, that means

- astronomical components of PM, associated with the multipole structure of the Earth’s inertia tensor (size up to 0.1 mas), should be considered by the S-WG 1 “Precession/nutation”;
- geophysical effects in nutation, mainly the FCN and S1 signals (size up to 0.5 mas), should be considered by the S-WG 2 “Polar motion and UT1”.

Contributions

Introduction

☒ Here, we focus on commenting two issues that, in our opinion, should be considered for discussion within SWG1 for future improvements of the models
☐ Direct effects of the rotation of the inner core
☐ Influence of the triaxiality of the Earth and its internal layers

Direct effects of the inner core (1)

☒ IAU 2000A nutation model just incorporates the inner core by considering its “indirect” effect. That is to say, by modifying the response-amplitudes to the “rigid” perturbing potential (reference configuration) via the associated new normal modes

\[ \frac{dH}{dt} + \Omega \times H = \Gamma \] (15a)
\[ \frac{dH_f}{dt} - \omega_f \times H_f = 0 \] (15b)
\[ \frac{dH_s}{dt} + \Omega \times H_s = \Gamma_s \] (15c)

The torque \( \Gamma \) on the Earth as a whole is that due to the degree 2 tesseral component \( \phi_s \) of the lunisolar perturbing potential:

\[ \Gamma = -\Omega_0^2 Ad \phi \times \phi \] (16a)

\textit{Mathews et al. 1991}
Direct effects of the inner core (2)

- However, the differential rotation of the solid inner core changes the matrix of inertia of the Earth, modifying the geopotential with respect to the "rigid" case.

- This fact has been previously considered by different authors (e.g., Greiner-Mai et al., 2000, 2001; Chen, Sen, & Han 2008) who studied the induced changes of polar motion and the gravity field.

- Its contribution to the nutational motion has been recently computed for a basic three-layers model (Escapa et al. 2012), providing some terms relevant at the μas level.

Direct effects of the inner core (3)

<table>
<thead>
<tr>
<th>Argument</th>
<th>Period</th>
<th>Figure axis (μas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_M$ $l_S$ $F$ $D$ $\Omega$ Days $\Delta \psi$ $\Delta \varepsilon$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0 +0 +0 +0 +1 -6793.48 2.79 -0.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0 +0 +0 +0 +2 -3396.74 0.00 -0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0 +1 +0 +0 +0 365.26 14.95 9.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0 -1 +2 -2 +2 365.25 -1.78 0.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0 +0 +2 -2 +2 182.63 44.61 -19.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0 +1 +2 -2 +2 121.75 1.64 -0.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+1 +0 +0 +0 +0 27.55 -2.22 0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0 +0 +2 +0 +2 13.66 7.17 -3.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0 +0 +2 +0 +1 13.63 1.22 -0.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+1 +0 +2 +0 +2 9.13 0.96 -0.41</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- For a consistent treatment of the influence of the inner core, should the direct effects be incorporated?
- As a correction or as a part of the precession-nutation model?
Triaxiality of the model (1)

- The triaxiality of the Earth, as a whole, provides short-period nutations worked out for rigid (e.g., Bretagnon et al. 1997, Souchay et al. 1999, Bizouard 2001) and non-rigid models (e.g., Getino et al. 2001, Brzeziński and Capitaine, 2003, Mathews and Bretagnon, 2003)

- Currently they are considered as diurnal and subdiurnal corrections (astronomical) to the motion of the CIP in the ITRS (IERS Conventions 2010, Brzeziński 2002), but they are not a part of IAU2000, based on an axial-symmetric Earth model

- The contribution of the triaxiality of the core to diurnal terms (Escapa et al. 2002, Mathews and Bretagnon 2003) is not taken into account in those corrections, since there was a large uncertainty in its value (about 2002)

Triaxiality of the model (2)

Table 5.1: Coefficients of sin(argument) and cos(argument) in $\Delta x \Delta y$ nutations, due to tidal gravitational (degree $n$) for a nonrigid Earth. Units are $\mu as$ and the expressions for the fundamental arguments (Dehantian arguments) are given by (40).

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\chi$</th>
<th>$\Delta$</th>
<th>$\Delta$</th>
<th>$\Delta$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0.55</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.65</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.65</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.65</td>
</tr>
</tbody>
</table>

IERS Conventions 2010, chap. 5
Triaxiality of the model (3)

- Chen & Shen (2010) have developed a SOS-like theory, considering from the beginning the effects of the triaxiality of the Earth and its core, and determining their values from new gravity models.

- They affect the nutations (long-period), since the triaxiality modifies the response of the Earth (e.g., through the expression of CW and FCN modes), changing the periods of the normal modes and providing new data fitting.

  - Should triaxiality be included in the precession-nutation model, at least regarding its indirect effect on nutations (long-period)?
  - Can we take currently a reliable value for the triaxiality of the core?
  - Should the corrections (astronomical) in polar motion be updated or also incorporated as a part of the precession-nutation model?
Juan Getino, Spain (getino@maf.uva.es)

Introduction

- **Main purpose:**
  - Carrying out a mathematical algorithm to improve the accuracy of the rotation Earth model by means of the correct integration of second order terms.

- **Guidelines: Hamiltonian formalism**
  - Application of powerful canonical integrations methods at second and higher orders.
  - Continuity with the study of rigid and non-rigid models.
  - Experience of the researcher group.

- **Previous work:**

- **Difficulties: Intrinsic complexity of second order solutions**
  - Rigid model: 6-dimensional phase space.
  - Two-layers model: 12 (24)-dimensional phase space.
  - Three-layers model: 18 (36)-dimensional phase space.

First Results

- **A New Formalism**
  - Based on the matrix formulation of equations of motion.
  - The dynamical variables of the considered model are gathered together in only two matrix variables, $X_0, X_1$, where $X_0$ is formed with the variables of small magnitude.
  - A Taylor expansion in powers of $X_0$ is performed, obtaining the dynamical equations in the form $\dot{X}_{0,1} = \sum_{\alpha=0}^{\infty} Y_{0,1}^{\alpha} X_0^{\otimes \alpha}$ as function of the matrices of the system $Y_{0,1}^{\alpha}(X_i; t)$.
  - These matrices $Y_{0,1}^{\alpha}$ characterize completely the considered Earth model.
  - A canonical perturbation method is applied to the generalized Hamiltonian $H(X_j, Z_j) = \sum_{j=0}^{1} Z_j^T X_j$, obtaining the second order analytical solutions as functions of matrix of the system.

- The solution is obtained in a systematic way.
- It is very simple to improve the model by considering new effects: only the modification of the system matrix $Y_{0,1}^{\alpha}$ is needed.
First Estimates

- Two-layers Earth model (Poincaré model)
  - Longitude in-phase of the figure axis spin-spin coupling second order contributions (μas):

<table>
<thead>
<tr>
<th>Period (days)</th>
<th>Rigid Earth</th>
<th>Two-layer</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6798.36</td>
<td>-30.21</td>
<td>-81.12</td>
<td>-50.91</td>
</tr>
<tr>
<td>305.20</td>
<td>1.05</td>
<td>-2.72</td>
<td>-3.77</td>
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<td>182.62</td>
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<tr>
<td>121.75</td>
<td>0.532</td>
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<td>27.55</td>
<td>0.14</td>
<td>-5.55</td>
<td>-5.69</td>
</tr>
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<td>13.66</td>
<td>-4.85</td>
<td>-51.23</td>
<td>-46.38</td>
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<tr>
<td>13.63</td>
<td>14.69</td>
<td>4.72</td>
<td>-9.97</td>
</tr>
<tr>
<td>9.13</td>
<td>0.04</td>
<td>-8.53</td>
<td>-8.57</td>
</tr>
<tr>
<td>Prec. (mas/cJ)</td>
<td>-46.24</td>
<td>-68.38</td>
<td>-22.14</td>
</tr>
</tbody>
</table>
José Manuel Ferrándiz, Spain  (jm.ferrandiz@ua.es)

Nutation-precession theories consistency

☐ IAU Precession-nutation model is comprised by two parts: IAU 2000A nutation (Mathews et al. 2002) and IAU 2006 precession (Capitaine et al. 2003).

☐ There are key constants and parameters common to both of them, like the dynamical ellipticity $H$, the value of the obliquity at the epoch $\varepsilon_0$, etc.

☐ The determination of their numerical values affects and is affected by any of the two parts, hence the need of ensuring consistency in the Earth models and parameters considered by them.

☐ Changes in the values of $\varepsilon_0$ and the J2 rate (IAU 2006) induce some corrections in the IAU 2000A nutation model (Capitaine et al. 2005, Escapa et al. 2013) (precession $\rightarrow$ nutation)

Nutation-precession theories consistency

☐ In the contrary sense, second order and tidal effects of the non-rigid Earth stemming from the nutation model contribute to the precession rates, affecting the value of $H$. So, the precession model is also affected by comple non-rigid nutation interactions (nutation $\rightarrow$ precession)

☐ Many of them are considered in IAU 2006, but not other like those arising from the second order core-core interactions (Ferrándiz et al. 2005 ) that provides a contribution about -12 567 $\mu$as/cy (Poincaré with elastic mantle model)

☐ It poses some questions about the more convenient way to ensure consistency between the precession and nutation theories, which is one of the main targets of the Working Group on Theory of the Earth Rotation
Precession nutation terminology


Nomenclature for the Current Precession and Nutation Models

Problem

For much of the last decade, we have described the latest and best adopted precession and nutation theories as the IAU 2000A Precession-Nutation Model, a phrase taken from IAU 2000 Resolution B1.6. Since the adoption of a new precession theory by the IAU in 2006, however, that phrase has gradually been abandoned without an agreed-upon replacement. The situation is complicated not just by the new precession theory, but by the availability of an adjusted IAU 2000A nutation model that is often not distinguished in print from the original. New agreed-upon nomenclature is required to avoid confusion.

Precession and Nutation Algorithms in Use Since 2000

The collection of algorithms used for precession and nutation over the past decade would include the following. I would argue that any combination of these precession and nutation models has a reasonable claim to be the “IAU 2000A Precession-Nutation Model”. In each description below, I give the justification for the idea that the particular algorithm could be considered part of that model.

For Precession:

P1: Corrections to delpsi and deleps, linear with respect to time, on top of the Lieske et al. (1977) precession formulation. Justification: This was the precession solution in the MHB nutation paper referred to in IAU 2000 resolution B1.6 (there listed as Mathews, Herring, and Buffet, 2000 – submitted to JGR). This is the consistent with what is stated in the IERS Conventions (2010) section 5.2.1.
P2: The interim IERS precession expressions given in the IERS Conventions (2003). Justification: IAU 2000 resolution B1.6 “encourages the development of new expressions for precession consistent with the IAU2000A model” and these expressions satisfy that mandate. They are provided in the IERS Conventions (2003) in section 5.5.2 as “Precession Developments Compatible with the IAU2000A Model” under the general heading of section 5.5, “IAU 2000A and IAU 2000B Precession and Nutation Model”.

P3: Capitaine et al. (2003) P03 precession, recommended by the IAU Working Group on Precession and the Ecliptic and adopted by the IAU in 2006. Justification: This is the culmination of the search for a new complete precession theory, compatible with the MHB corrections for precession, that began shortly after the 2000 General Assembly. It is now the IAU recommended model.

For Nutation:


N2: The MHB (2002) nutation series with corrections for P03 precession. Justification: The small additional corrections are needed for the IAU adopted nutation and precession to be consistent at the highest levels of precision.

These are the basic theories. We also have subsidiary data, such as the series for the X and Y of the pole, the “complementary terms”, and the series for $s$, derived from them.

I suspect that just about all combinations of these algorithms are in use, or have been in use over the past decade, with everyone believing they are implementing “IAU 2000A”. Perhaps so — it could be reasonably argued that at the level of current observations, it doesn’t really matter. Although that may be true from a scientific point of view, it does not satisfy many software developers who want to do things canonically, and have a requirement to use the latest and best IAU/IUGG/IERS models. They have seen the phrase “IAU 2000A Precession-Nutation Model” used a lot, and want to know exactly what it means. Even the current SOFA routine names are a bit confusing in this regard, especially
to outsiders. It is also important for us in reading the literature, evaluating data, and conducting numerical tests that we know exactly what algorithms have been used.

The IAU Working Group on Nomenclature for Fundamental Astronomy provided the following definition in 2006:

**IAU 2000 precession-nutation:** *IAU Resolution B1.6 recommends the IAU 2000A precession-nutation model for those who need a model at 0.2 mas level. It represents the CIP. An abridged model, designated IAU 2000B, is available for those who require a model at the 1 mas level.*

However, given the number of algorithms to choose from, this definition does not provide the needed specificity.

**Recent Descriptions of the Models in Use Now**

In the IERS Conventions (2010), the phrase “IAU 2000A Precession-Nutation Model” has disappeared (as far as I can tell) as a description of the current algorithms. In section 5.2.2 it is stated, “The IAU precession-nutation model resulting from IAU 2000 Resolution B1.6 and IAU 2006 Resolution B1, will be denoted IAU 2006/2000A precession-nutation in the following.” (This would be the P3+N1 combination.) In 5.6.1, it is noted that the IAU 2000A nutation is also denoted MHB2000. In 5.6.3, the adjustments to the IAU 2000A nutation are described (the nutation with adjustments is N2 above) and the recommended notation for the adjusted nutation series is IAU 2000AR06.

SOFA refers to P1+N1 as the “00A” model, and P3+N2 as the “06A” model. One of SOFA’s web pages contains a note that says “06A” is a shorthand for “IAU 2006+2000A precession-nutation” but that it does not imply the existence of an IAU 2006A nutation. Another paragraph on the same page explains that there is a transformation that has been applied to the IAU 2000A nutation in order to make it compatible with the IAU 2006 precession.
However, IERS Bulletin A says that the dX and dY Celestial Pole Offsets are referred to the “IAU2000A Nutation/Precession Theory”, which as we’ve seen is quite ambiguous. Perhaps in the case of Bulletin A the ambiguity is deliberate. I’ve been told the data there actually reflect a combination of whatever the individual analysis centers are using, which is not tightly controlled. The language at the bottom of Bulletin A has not changed since 2005 (which is the earliest version still on the IERS web site). Obviously there have been, or should have been, some software transitions since then, so it appears that the phrase “IAU2000A Nutation/Precession Theory” may be used in Bulletin A in a generic sense.

Since the 2009 edition, *The Astronomical Almanac* explains things as follows (page B2, near bottom): “The 2006 IAU General Assembly adopted various resolutions, including the recommendations of the Working Group on Precession and the Ecliptic (WGPE), which in this section are designated IAU 2006... It should be noted that the IAU 2006 precession parameters are to be used with the IAU 2000A nutation series. However, for the highest precision, adjustments are required to the nutation in longitude and obliquity (see page B55). These adjustments are included in the fourth release of the IAU SOFA code which is used throughout this section.” That is, in Section B, Time Scales and Coordinate Systems, the computations are done with P3+N2 = SOFA’s “06A” model. However, in some other sections of *The Astronomical Almanac*, P3+N1 is used. There is a similar statement on pages L1-L2, and, like the IERS Conventions, the almanac has abandoned the use of the phrase “IAU 2000A Precession-Nutation Model” when referring to the data in the most recent editions.

I would probably argue that until 2006, P2+N1 defined the IAU 2000A model, and since 2006, P3+N1 defines it (or possibly P3+N2, although the adjustments in N2 were never addressed by the IAU). I think it is incorrect to say that the IAU 2000A precession can *only* refer to the original MHB solution for linear-with-time corrections to delpsi and deleps (the P1 algorithm), because it ignores the stated intent of Resolution B1.6 that new expressions for precession be developed. It also ignores seven years of IERS practice that the P2 algorithm is an implementation of IAU 2000A precession.

**Recommendation**

For the future, in order to try to accommodate as much as possible the phrases that are already in use, but to make things more specific, I would propose the following definitions:

**IAU 2000A Precession-Nutation Model**: A generic reference to any pair of algorithms, one for precession and one for nutation, that together correctly implements the model of the motion of the CIP in the GCRS given in Mathews, Herring, and Buffet (2002, JGR, the
basis of IAU 2000 Resolution B1.6) at the accuracy of the best observations available near the year 2000, about 0.2 mas (ignoring the free core nutation).

In the following, I suggest using the term “algorithm” instead of “model” to denote particular mathematical developments that implement the same physical model.


**IAU 2006/2000A\textsubscript{R06} Precession-Nutation Algorithm or**

**IAU 2006/2000A(R06) Precession-Nutation Algorithm or**

**IAU 2006/2000A\textsuperscript{*} Precession-Nutation Algorithm or**

**IERS (2010) Precession-Nutation Algorithm:**

A specific implementation of the IAU 2000A Precession-Nutation Model using the Capitaine et al. (2003) P03 precession (also described in Hilton et al. 2006), implemented in any set of angular parameters, and the Mathews, Herring, and Buffet (2002) nutation series as adjusted for compatibility with the P03 precession.

This phrase describing the latter concept could also be simplified to the **IAU 2006 Precession-Nutation Algorithm** if we agreed that the adoption of the P03 precession development by the IAU in 2006 implied the need for corrections to the MHB nutation series. That seems to me to be somewhat a re-write of history, as I do not recall any discussions to that effect in Prague. At this late date, it also might cause some confusion. However, it would simplify the terminology!

Certainly other terminology would be possible and other suggestions are invited. The idea is simply to standardize our terminology to ensure that we understand specifically what algorithms are being used by whom in what circumstances.

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